

Simulating the Performance Variability matrix using Markov chains

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Abstract

In Operations performance analytics space, specifically into financial services firm we measure the associate or functional performance in form of performance rate which is the magnitude of requests processed per unit of time. Utilization is another key metrics that measures the time spent on core processing work as a percentage of the total time spent on core and the non-core work. Basis these key KPIs in relation to the variability of performance we map the associates into 4 performance states where each state provides the unique performance insight to line managers for better planning. The assumption here is that the performance rate and utilization levels at time “t” are independent of the past and only depend on the “t-1” state. Hypothesis for the same is that every interval of time may have different work complexity and business environment under which financial operations would function and thus assuming the same to be stochastic in nature. We then use the Markov chain application to make relevant business inferences and forecasts for the defined performance states.

Modeling approach:

(Data is hypothesized for illustration purpose & tool used is R)

Keywords: Operations, risk, Performance, optimization, Markov chain, analytics, Finance

1) Introduction

Let $V_1, V_2, V_3, \dots, V_n$ be the individual customer requests been processed by an individual associate X_1 where the time taken to resolve the volumes are $t_1, t_2, t_3, \dots, t_n$ then

Performance rate $R_{x1} = \sum_{i=1}^n V_i / \sum_{i=1}^n t_i$

$t_1, t_2, t_3, \dots, t_n$ Also defines the time spent on actual core work by the associate while $t_{c1}, t_{c2}, t_{c3}, \dots, t_{cn}$ defines the time spent on non-core activities

Utilization $U_{x1} = \sum_{i=1}^n t_i / (\sum_{i=1}^n t_i + \sum_{i=1}^n t_{ci})$

Basis the above two metrics we then define the compound metrics “Performance score” as

$$P_{x1} = R_{x1} * U_{x1}$$

If the Performance rate of an associate is high but the utilization levels are low it highlights that the performance capability is not adequately utilized. The ideal scenario here would be the high performance rate accompanied with the adequate utilization levels.

It becomes imperative for the line managers to ensure that their performance scores are improving consistently over the time period. Any inconsistency or high level of variability in the performance scores can compound to operational risk for the organization.

Variability in the performance score of an associate is defined as $\sigma_{P_{X1}}$ for an individual associate X_1 and can result from internal & external business factors and management needs to be vigilant about the same. Basis the two metrics P_{Xn} & $\sigma_{P_{Xn}}$ we define four possible states into which the associates would fall.

These are summarized below:

Performance state	Description
A	High P_{Xn} and Low $\sigma_{P_{Xn}}$
B	High P_{Xn} and High $\sigma_{P_{Xn}}$
C	Low P_{Xn} and Low $\sigma_{P_{Xn}}$
D	Low P_{Xn} and High $\sigma_{P_{Xn}}$

Table: 1

The optimal scenario would be the high performance score and low variability in the same. Any increase in the variability of the performance scores should call for a deep dive by the frontline managers to investigate the reasons for the same and take corrective measures.

Economic and business scenarios are more dynamic in nature due to changing regulations, client requirements, high competition in the industry, changing technology and demographics. Thus the way financial operations would run in a given time scenario would be different from how it used to function historically. This forms our basis to the assumption that Operational KPI's are independent to the historical observations and can only be at best related to the business environment at time " $t - 1$ ".

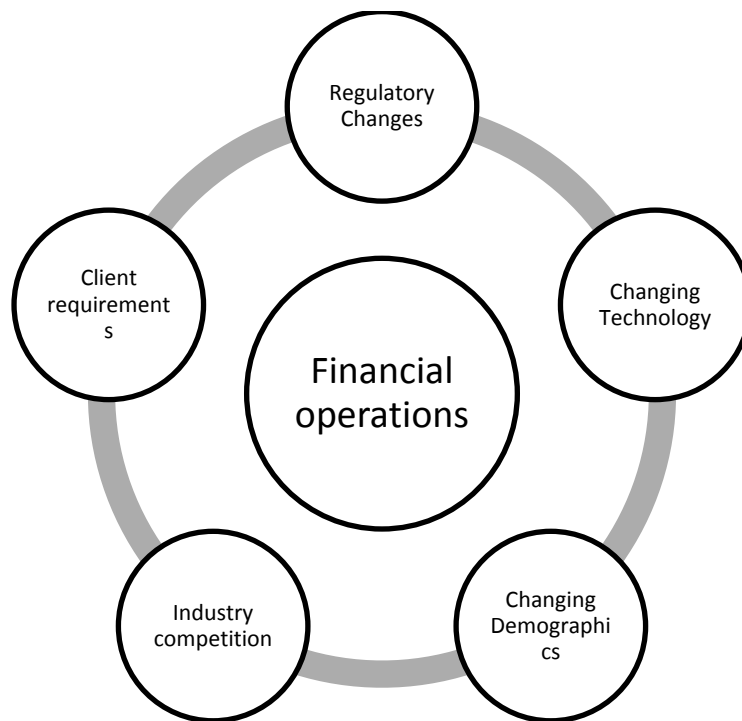


Fig: 1

2. Performance Score Vs Variability Framework: PV Matrix

Data table structure

Employee	Quarter	Performance Score	Variability of Performance Score
X_1	Q1	7	30%
X_2	Q2	8	55%

Table: 2

PV Matrix highlights 4 scenarios in form of quadrants characterized as A, B, C and D. Quadrant A denotes the optimal scenario with high performance score and low variability while quadrant D is the most risky zone where the associate performance scores are low accompanied with high level of variability. The line managers should work towards ensuring that maximum percentage of their associates would fall under quadrant A and enabling their team members to migrate out from quadrant D and C.

Performance Score Vs Variability framework

Performance Mix

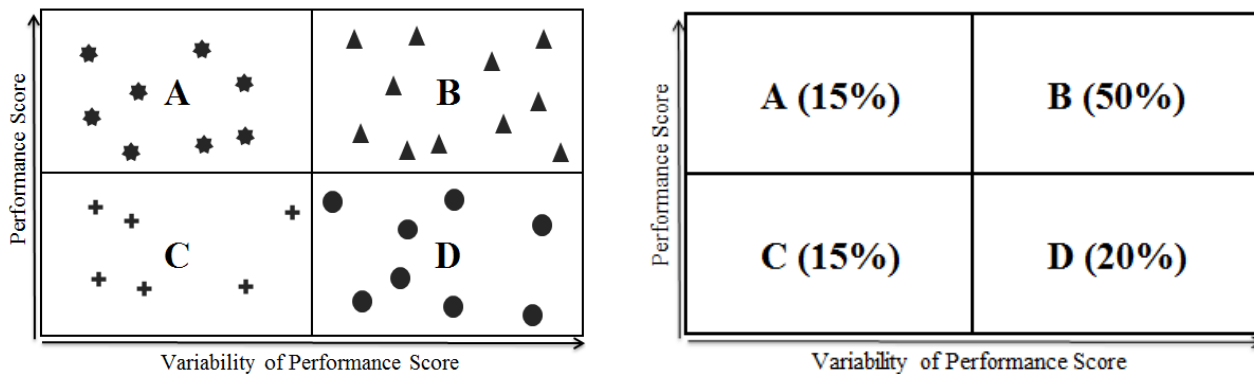


Fig: 2

The above composition highlights the initial state of the PV Matrix with 15% of the associates falling in quadrant A, 50% of the associates under quadrant B, 15% of the associates under quadrant C and 20% of the associates falling under the risky zone i.e. quadrant D. Quadrant D should be the core area of focus for the management to improve upon.

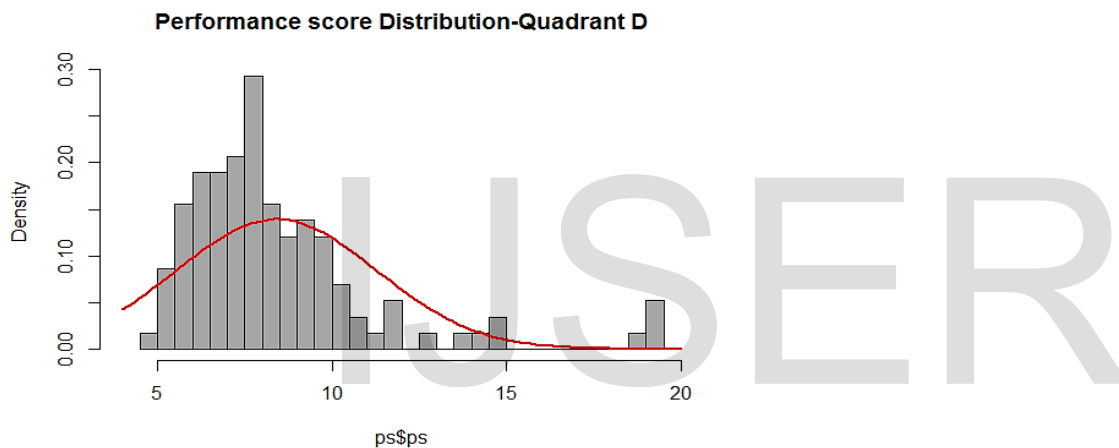


Fig: 3

Distribution of Utilization (X axis), Variability (Y axis) & Performance Rate (Z axis) for quadrant D associates

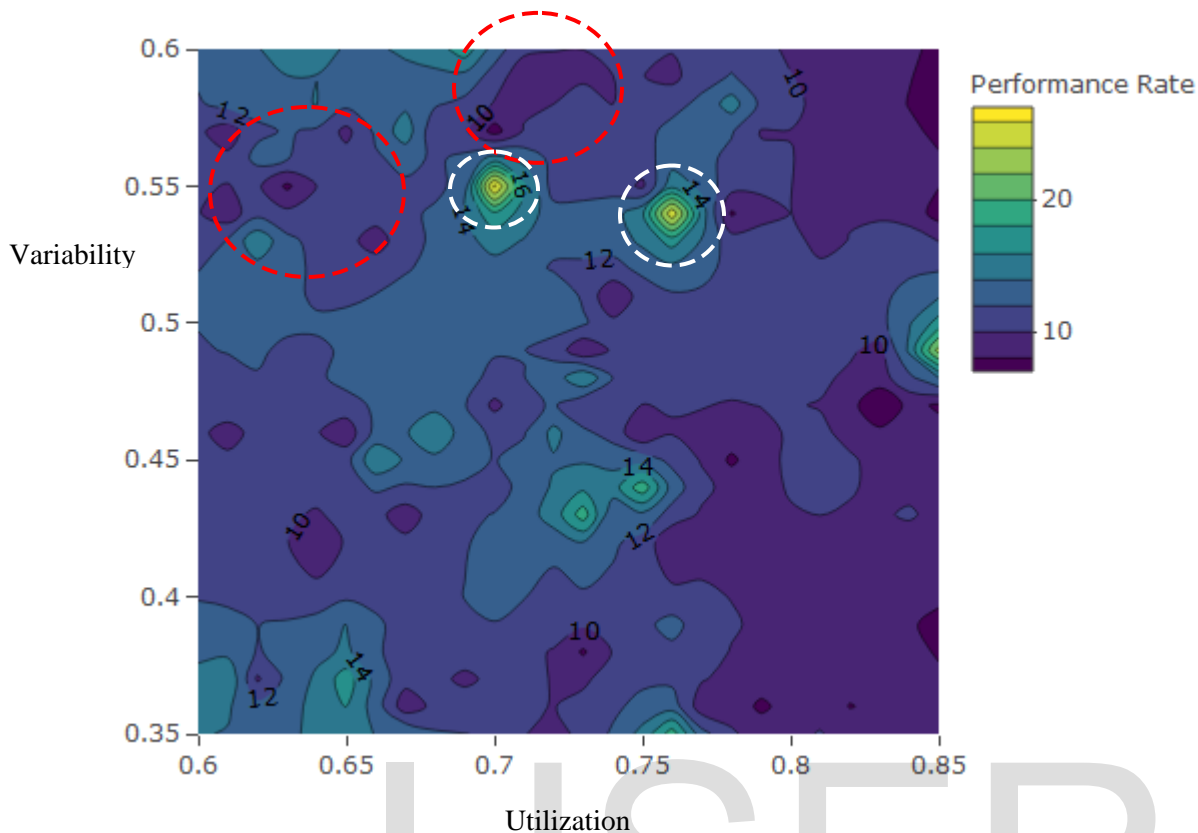


Fig: 4

The Zones highlighted under the red circle are the ones that require immediate attention from the line managers. These points highlight the case of high variability, low Utilization levels and very low Performance Rate. Zones highlighted under white circle are running around the threshold Performance rate level or probably better thus here the focus is to bring down the variability of the performance score which is a compound metrics derived from U_{xn} and R_{xn} . Given the initial state of the PV Matrix we can derive business inferences by forecasting the future state of the above Performance mix.

3. Application of Markov chains to PV Matrix.

A Stochastic process is said to follow a Markov property if the conditional probability distribution of random process “future state” depends on the “current state” and not on the preceding events.

Statistical theory: If we have a sequence of stochastic variables $K_1, K_2, K_3 \dots K_n$ then as per Markov property the probability distribution of the state K_{n+1} depends upon the initial state of K_n and not the preceding states $K_{n-1}, K_{n-2}, \dots, K_{n-k}$

$$\Pr(K_{n+1} = k_{n+1} | K_n = K_n)$$

The transition of random variable chain from one state X_i to another state X_j is denoted by P_{ij}

$$P_{ij} = \Pr(K_1 = X_j | K_0 = X_i)$$

If the migration probabilities of transitioning from one state to the other remain fixed with the number of time steps it reflects the Time homogeneity property of the Markov chain.

$$\Pr(K_{n+1} = X_j | K_n = X_i) = \Pr(K_n = X_j | K_{n-1} = X_i)$$

In our “Performance score Vs Variability framework” we have defined the associate performance into four possible states in a two dimensional graph.

$$\{X_1 = A, X_2 = B, X_3 = C, X_4 = D\}$$

If all the possible states in the framework communicate with each other, $X_i \rightarrow X_j$ and $X_j \rightarrow X_i$ then the Markov chain is said to be irreducible. An absorbing state is where the transition probability of a particular state to itself is 1 thus $P_{jj} = 1$ for $X_i \rightarrow X_i$

We will use the “markovchain” package in R to simulate the future state of the framework

```
> library("markovchain")
> performancestates=c("A","B","C","D")
```

Basis the graphical representation of the framework the performance states in the above function have been characterized as A, B, C and D

Performance migration matrix: Basis the historical research and performance trends of the associate’s journey “performance migration matrix” is derived. It highlights the probability of the associates transitioning from one quadrant to the other and gives the scenarios of how our future state will evolve going ahead in time.

Table 2: Performance Migration matrix (Quarterly)

	A	B	C	D
A	55%	25%	10%	10%
B	20%	60%	15%	5%
C	10%	30%	50%	10%
D	10%	20%	30%	40%

Table: 3

Associates falling into quadrant A have 25% probability of transitioning to quadrant B, 10% probability each of transitioning to quadrant C & D. for the associates under quadrant D which stands to be the risky zone for the financial operations there is a high probability of 40% that they would continue to remain under the same quadrant while they also have a high probability of 30% of moving into quadrant C.

Associates falling under quadrant C are generally new hires who in the initial state would be under the learning curve and thus have a high probability of remaining under quadrant C over the next quarter. Performance migration matrix needs to be updated on a timely basis (generally yearly) to realize the transition probabilities among the quadrants. For our future state analysis purpose we will assume the same to be constant over our period of analysis.

Assuming the migration probabilities to be constant over time can pose risk to the organization. The reason for the same is that the business dynamics may change over time which can have a direct impact on the work mix and complexity of the client requests.

Threshold levels of performance score and Variability in performance scores used to derive the quadrants A, B, C and D should be updated on a yearly basis for the model to reflect the realistic scenario.

Markov chain has its applications across various domains and is highly used in the credit risk modelling space where transition probabilities are updated by the rating agencies on yearly basis. It is used to make inferences around default risk of a firm or a country.

>

```
performancematrix=matrix(data=c(0.55,0.25,0.1,0.1,0.2,0.6,0.15,0.05,0.1,0.3,0.5,0.1,0.1,0.2,0.3,0.4),byrow=TRUE,nrow=4,dimnames=
=list(performancestates))
```

Above matrix function under "markovchain" package is used to create the performance migration matrix.

```
> PS= new("markovchain", states = performancestates, byrow = TRUE,transitionMatrix = performancematrix, name =
"Performance")
```

```
> summary(PS)
```

Summary function provides the detail for closed classes, recurrent classes, Transient classes and absorbing states.

Performance Markov chain that is composed by:

Closed classes:

A B C D

Recurrent classes:

{A,B,C,D}

Transient classes:

NONE

The Markov chain is irreducible

The absorbing states are: NONE

Graphical representation of the Migration matrix

```
> plot(PS, edge.arrow.size=0.9,col="red",lwd=10)
```

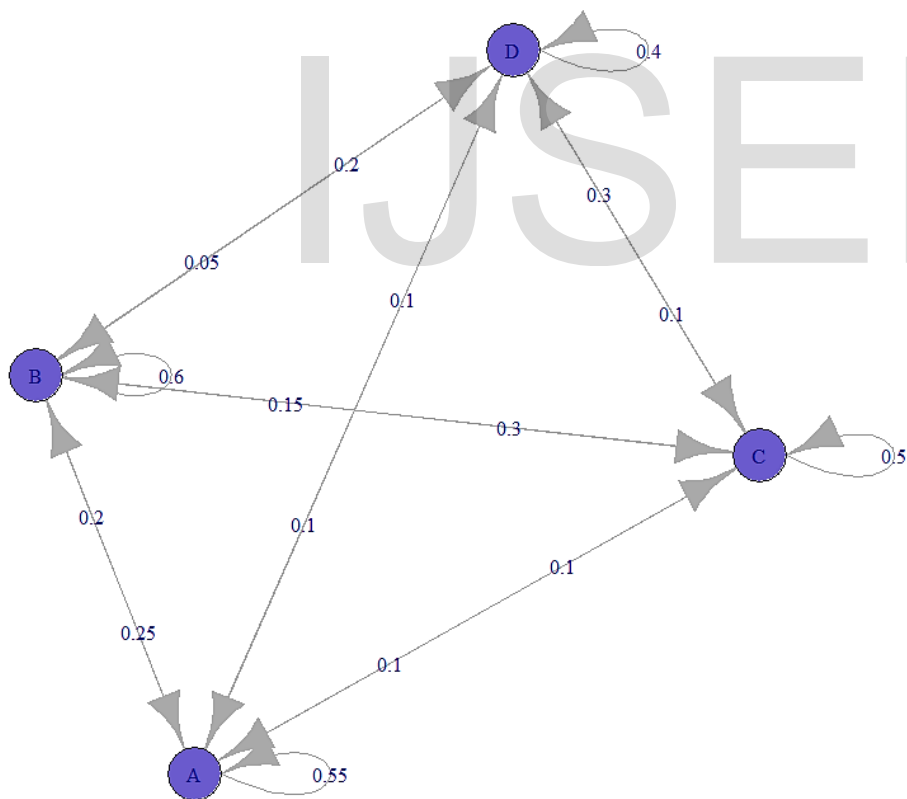


Fig: 5

```
> initialstate=c(0.15,0.5,0.15,0.2)
```

Basis the Performance migration matrix and initial state of the PV Matrix we can make business inferences about the future state of the framework.

> $\text{after1quarter} = \text{initialstate} * (\text{PS})$

> after1quarter

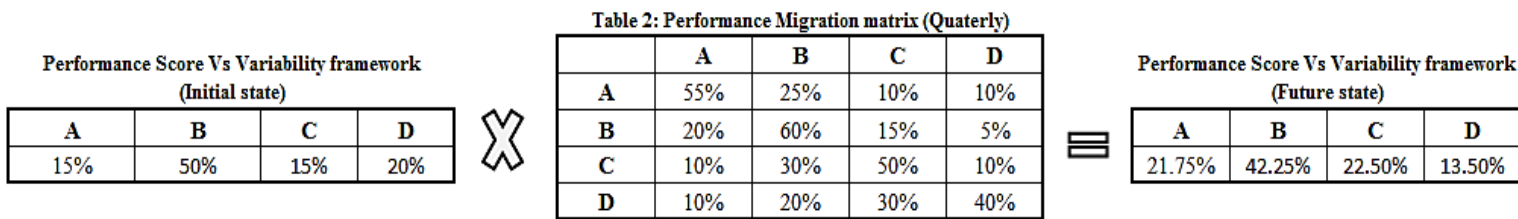


Fig: 6

One quarter from now we can expect the quadrant A of the "PV Matrix" to have 21.75% of the associates falling under the same, 42.25% under quadrant B while quadrant C & D having 22.50% and 13.50% of the total population.

Similarly we can derive the future state till infinity assuming the migration matrix to be constant (though not a realistic assumption).

Future state after 2 quarters: > $\text{after2quarters} = \text{initialstate} * (\text{PS}^2)$

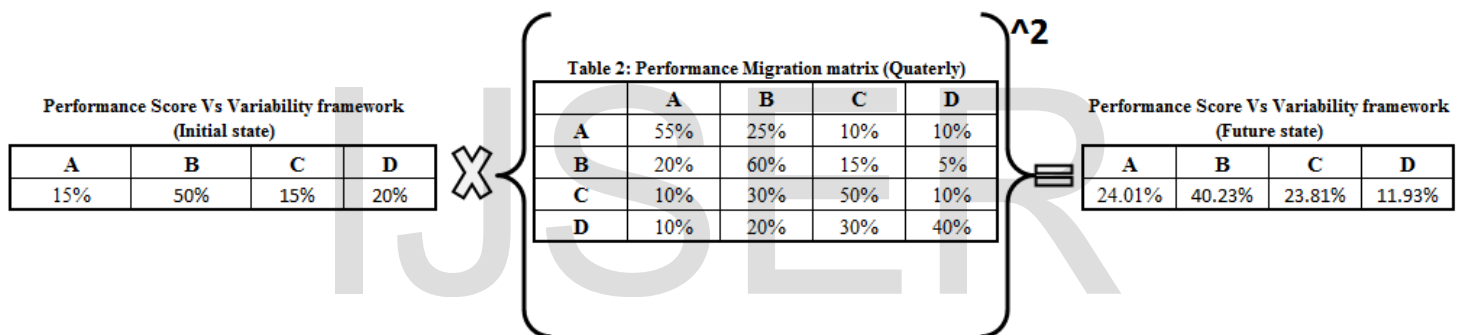


Fig: 7

The above calculation gives the Associate mix in the quadrants of the PV Matrix after 2 quarters of timeframe.

Future state after 20 quarters: > $\text{after20quarters} = \text{initialstate} * (\text{PS}^{20})$

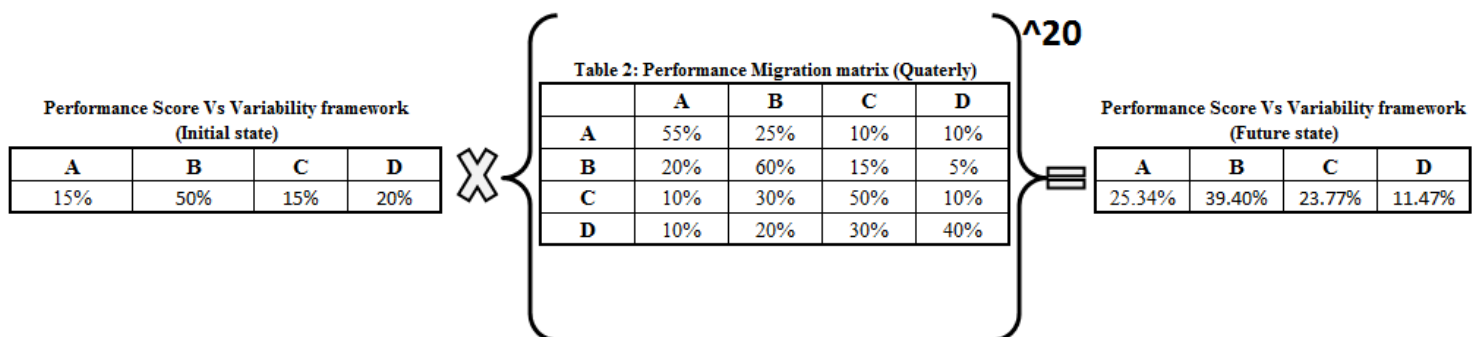


Fig: 8

Business inferences from the above migration matrix can be derived basis the probability theory. Let's say for example that we want to know about the probability that an associate who is currently into quadrant C will move to quadrant A in a year's time.

> (PS^4)

Performance^4

A 4 - dimensional discrete Markov chain defined by the following states:

A, B, C, D

The transition matrix (by rows) is defined as follows:

	A	B	C	D
A	0.2727063	0.3843563	0.2259688	0.1169688
B	0.2584000	0.4023750	0.2294750	0.1097500
C	0.2365375	0.3952875	0.2530625	0.1151125
D	0.2291125	0.3845625	0.2604125	0.1259125

From the above result we can infer that the probability of an associate moving from quadrant C to quadrant A in a year's time is 23.65% and also there is 11.51% probability of the associate migrating to quadrant D over 1 year. Below chart describes the risk of an associate transitioning from Quadrant A to Quadrant D. The probability increases till the 3rd quarter and then stagnates to 11.50% levels from Q5.

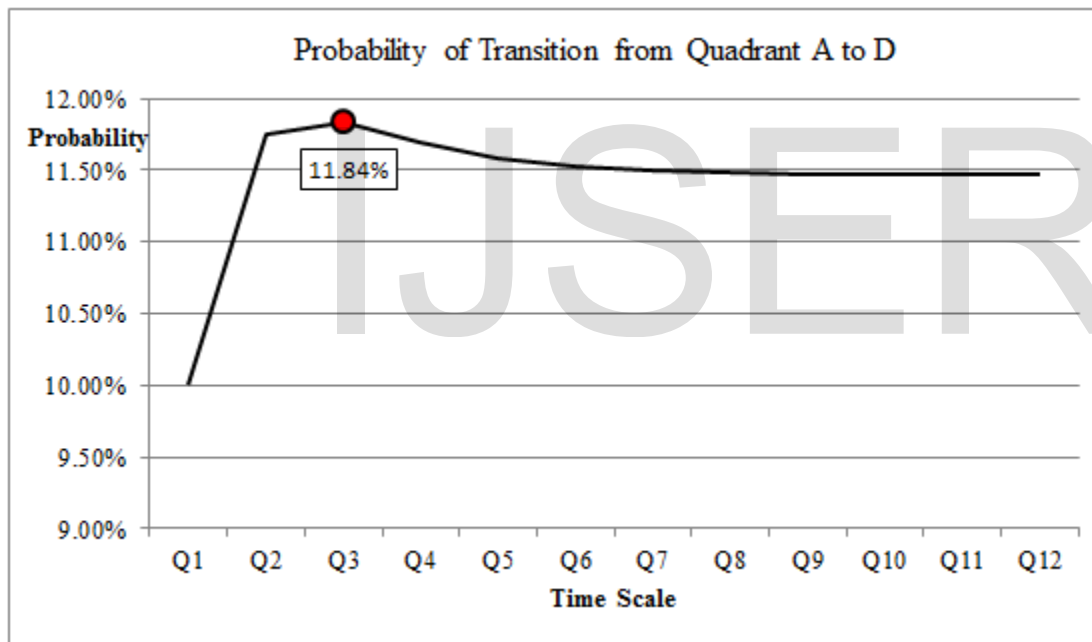


Fig: 9

4. Conclusion

The Primary objective of the paper is to describe how a framework can be designed using the key performance KPI's into operations analytics like Performance rate, Utilization and variability as a risk measure to describe the performance states. High level of variability in the performance scores should make the concerned line managers to deep dive into the factors accompanying the same. Basis initial performance states the future state of the PV Matrix can be derived using Markov chain technique. Key here for the business is to focus on the quadrants C and D that pose major risk to operational performance of the

organization and if the future states contain significant portion of the population percentage in the risky quadrants then it calls for the relevant corrective measures for the frontline managers to improve the same.

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